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Optimal Strategies for Free-Flight Air Traffic Conflict Resolution

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Recent advances in navigation and data communication technologies make it feasible for individual aircraft to plan and fly their trajectories in the presence of other aircraft in the airspace. This way, each aircraft can take advantage of the atmospheric and traffic conditions to optimally plan its path. This capability is termed the free-flight concept. Whereas the free-flight concept provides new degrees of freedom to the aircraft operators, it also brings in complexities not present in the current air traffic control system. In this concept, each aircraft has the responsibility for navigating around other aircraft in the airspace. Although this is not a difficult task under low-speed, low-traffic-density conditions, the complexities of dealing with potential conflicts with multiple aircraft in other flight conditions can significantly increase the pilot's work load. The development of conflict resolution algorithms is presented based on trajectory optimization methods that will the enable the practical implementation of the free-flight concept. These algorithms use nonlinear point-mass aircraft models and include realistic operational constraints on individual aircraft. Several conflict resolution scenarios are illustrated. The present analytical framework can also incorporate information about ambient atmospheric conditions. These conflict resolution algorithms are suitable for implementation onboard aircraft.

I. Introduction

RECENT advances in global positioning system-based navigation techniques and satellite data communication technology have motivated the Federal Aviation Administration (FAA) and commercial air traffic carriers to consider the free-flight concept as a major component of the future air traffic control system.[1,2] Free flight is defined as safe and efficient flight operations under instrument flight rules in which the operators have the freedom to select their path and speed in real time. Thus, individual aircraft will be able to plan and execute their trajectories without direction from any external agents during most of their flight duration. The ability of individual aircraft to plan their trajectories can result in substantial improvements in operational efficiency because the aircraft can take advantage of atmospheric and traffic conditions to minimize delays and fuel consumption. This approach can also lead to optimal coordination between aircraft involved in hub/spoke modes of operation currently employed by most major air carriers. However, because different aircraft in a given airspace may have conflicting objectives, the trajectories synthesized by individual aircraft can generate conflicts with other aircraft trajectories. Successful implementation of the free-flight concept will require the development of systematic methods for multiaircraft conflict resolution. Whereas conflict resolution strategies are obvious under low-speed, low-traffic-density conditions, the problem can become complex as the traffic density and the aircraft speeds increase. This fact has motivated a number of recent studies on various aspects of the free-flight conflict resolution problem.[3–12]

In Ref. 7, optimal control theory was used to maximize the range at the point of closest approach and dynamic programming was used to obtain solutions. The authors examined conflict resolution between two aircraft and considered heading and speed changes separately. They developed a set of charts, which could be used to determine rules of the road for an arbitrary initial condition. Reference 3 evaluated altitude changes, speed changes, and heading changes separately to determine the effectiveness and warning time necessary for each type of maneuver. Only two aircraft were involved in each case.

A genetic algorithm was used to resolve multiple-aircraft conflicts in Ref. 10. Predefined maneuvers were used to construct the new trajectories. The study focused on minimizing delays, maneuver durations, and the number of maneuvers in two dimensions. The approach in Ref. 10 requires simulating the aircraft trajectories, but the aircraft models were not discussed, and it does not appear that constraints such as load factor or fuel consumption were included. Although the authors[10] claimed to have achieved globally optimal solutions through the use of genetic algorithms, they did not provide any proof that a global minimum was indeed achieved.

A self-organizational approach to the conflict resolution problem was advanced in Ref. 11. In that work, each aircraft is assumed to behave like a positively charged particle, and the destinations are assumed to be negatively charged. Separation between aircraft is primarily facilitated by the repulsion between like charges assumed to be carried by individual aircraft. Speeds were assumed constant, and only two-dimensional conflict resolution scenarios were considered. No attempt was made to model the dynamics of the aircraft or to incorporate practical constraints such as load factor limits or fuel consumption. The idea of potential and vortex fields used in robot collision avoidance forms the basis for the work reported in Ref. 12. Two-aircraft conflicts are handled as a combination of overtake and head-on maneuvers. Multiple-aircraft conflicts are handled with a round-about maneuver. Again, results are given for conflict resolution in two dimensions. In this formulation, individual field strengths are adjustment parameters. Note that the approach cannot be considered as optimal in any sense.

The focus of the research effort described in this paper is on the development of systematic methods for multiple-aircraft conflict resolution. Conflict resolution algorithms employed in the present air traffic management automation tools are largely rule based and are designed to operate under the existing air traffic control procedures.[13–17] Whereas these methods are adequate for resolving simultaneous conflicts between aircraft following standard jet routes, it is not clear how these methods can be generalized to handle multiple-aircraft conflicts that can arise in the free-flight environment. The emphasis of the research reported in the present paper is on formulating and solving the conflict resolution problem as a trajectory optimization problem. As with other aerospace

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guidance problems, application of optimal control theory[18] and differential game theory[19] will result in more systematic methodologies for the solution of the conflict resolution problem. This has been demonstrated in several recent aerospace trajectory synthesis problems.[20–24] Unlike rule-based conflict resolution approaches, one of the advantages of the formalism presented here is that the algorithms are independent of the conflict resolution geometry. Present research builds on a previous research effort[5,25] on the conflict resolution problem.

Conflict resolution methodologies developed in the present research assume that the nominal trajectory of each aircraft trajectory involved in a potential conflict is given by a sequence of four-dimensional (three position coordinates and time) waypoints. Nominal waypoint sequences are adjusted by the conflict resolution algorithm to synthesize conflict-free trajectories that optimize desired performance indices. The conflict resolution problem is formulated both as a single-objective optimization problem and as a multiple-objective optimization problem.[26,27] Interaircraft conflict is defined through a conflict envelope in the form of an oblate spheroid. The trajectory optimization formulation uses nonlinear point-mass models of aircraft with realistic aerodynamic and engine models.[28,29] Two different cost functions are used in this study: a linear combination of total flight time and fuel consumption and an integral square of the perturbation from nominal trajectories. Vehicle models, constraints, and trajectory parameterization schemes are discussed in Sec. II.

Iterative numerical methods and a semianalytical guidance law are developed based on trajectory optimization theory.[18] Iterative methods that formulate the conflict resolution problem as a single-objective optimization problem and as a multiple-objective optimization problem are given in Sec. III. Semianalytical guidance law development using the neighboring extremal theory is presented in Sec. IV. Several conflict resolution scenarios are given in Secs. III and IV to illustrate the performance of these algorithms. In every case, it is shown that the present methods produce useful conflict resolution strategies. Furthermore, the results show that the conflict resolution algorithms developed are general enough to handle every type of conflict that can occur in the free-flight air transportation environment. Conclusions from the present research effort and future research directions are given in Sec. V.

II. Aircraft, Trajectory, and Conflict Envelope Models

Three major components of the conflict resolution problems analyzed in the present research are the aircraft models, trajectory parameterization schemes, and conflict envelope definitions. Efficiency of the conflict resolution algorithms can be greatly enhanced by using appropriate aircraft models and trajectory parameterization schemes. The computational efficiency of implementing aircraft operational constraints can be improved by employing appropriate transformations on the aircraft models. Careful definition of the interaircraft conflict envelopes can further improve the conditioning of the conflict resolution problem. Each of these issues is discussed in the following subsections.

A. Aircraft Models

Point-mass aircraft models are used in the present research. The point-mass aircraft model captures most of the dynamical effects encountered in civil aviation aircraft. These models are popular in aircraft performance work and are used in practically every aircraft trajectory optimization problem discussed in the literature. The point-mass equations of motion are formulated with respect to a coordinate system shown in Fig. 1.

The point-mass model assumes that the aircraft thrust is directed along the velocity vector and that the aircraft always performs coordinated maneuvers. It further assumes a flat, nonrotating Earth. These assumptions are reasonable for civil aviation aircraft operating within a range of 200 nautical miles. Since the conflict resolution generally occurs within this range, the fidelity provided by the point-mass model is adequate for formulating these problems. Note that it is feasible to develop point-mass models valid for spherical Earth approximations also. The point-mass model equations describing aircraft flight are

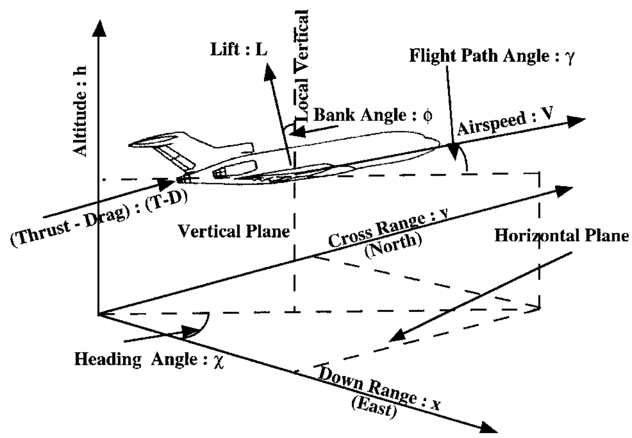

Fig. 1 Aircraft coordinate system.

$$\dot{V}_i = \frac{(T_i - D_i)}{m_i} - g \sin\gamma_i, \qquad \dot{\gamma}_i = \frac{g}{V_i}\left(\frac{L_i \cos\phi_i}{g\, m_i} - \cos\gamma_i\right) \tag{1}$$

$$\dot{\chi}_i = \frac{L_i \sin\phi_i}{m_i V_i \cos\gamma_i}$$

$$\dot{m}_i = -Q_i \tag{2}$$

$$\dot{x}_i = V_i \cos\gamma_i \cos\chi_i, \qquad \dot{y}_i = V_i \cos\gamma_i \sin\chi_i, \qquad \dot{h}_i = V_i \sin\gamma_i \tag{3}$$

with $i = 1, 2, \ldots, n$ being the aircraft under consideration. In these equations, V_i is the ground speed and is assumed to be equal to airspeed. T_i is the aircraft engine thrust, D_i is the drag, m_i is the aircraft mass, g is the acceleration due to gravity, γ_i is the flight-path angle, L_i is the vehicle lift, ϕ_i is the bank angle, χ_i is the aircraft heading angle, x_i is the down-range displacement, y_i is the cross-range displacement, h_i is the altitude, and Q_i is the fuel flow rate. The fuel flow rate depends on the altitude, Mach number, and the engine thrust. The aircraft drag is given in terms of the zero-lift drag coefficient and induced drag coefficient. Both of these are specified as functions of Mach number and the aircraft configuration settings such as the deployment of flaps, speed brakes, and the landing gear. For the present research, it is assumed that the aircraft is in the cruise flight configuration. The control variables in the aircraft model are the load factor n_i, controlled using the elevator; the bank angle ϕ_i, controlled using a combination of rudder and ailerons; and the engine thrust T_i, controlled using the throttle. Throughout the conflict resolution process, the control variables will be constrained to remain within specified limits.

For analytical convenience, the kinematic equations describing the aircraft position can be differentiated once with respect to time, and the remaining dynamic equations can be used to cast the point-mass aircraft model in an alternate form as

$$\ddot{x} = U_1, \qquad \ddot{y} = U_2, \qquad \ddot{h} = U_3 \tag{4}$$

where U_1, U_2, and U_3 are the three new control variables in the point-mass model. The relationship between these control variables and the actual control variables is given by the expressions[30]

$$\phi = \tan^{-1}\left[\frac{U_2 \cos\chi - U_1 \sin\chi}{\cos\gamma(U_3 + g) - \sin\gamma(U_1 \cos\chi + U_2 \sin\chi)}\right] \tag{5}$$

$$n = \frac{\cos\gamma(U_3 + g) - \sin\gamma(U_1 \cos\chi + U_2 \sin\chi)}{g \cos\phi} \tag{6}$$

$$T = [\sin\gamma(U_3 + g) + \cos\gamma(U_1 \cos\chi + U_2 \sin\chi)]m + D \tag{7}$$

The heading angle χ and the flight-path angle γ are computed as

$$\tan\chi = \dot{y}/\dot{x}, \qquad \sin\gamma = \dot{h}/V \tag{8}$$

This latter form of aircraft point-mass model is useful for mapping geometric trajectory parameters in terms of the aircraft control variables and vice versa. Thus, constraints on the aircraft control variables can be readily transformed into constraints on the trajectory curvature. Such mappings simplify the computations and are useful for efficient implementation of control constraints.

B. Trajectory Parameterization

Trajectory parameterization methods allow the description of individual aircraft trajectories using a small number of parameters. In addition to providing a compact description of the trajectories, trajectory parameterization methods permit the solution of the trajectory optimization problems as parameter optimization problems. Parameterized aircraft trajectories are described using a set of four-dimensional waypoints, consisting of three position components and time, together with the specification of the type of curves joining them. A piecewise linear trajectory parameterization is highly efficient from a computational standpoint. However, the piecewise linear trajectory parameterization implies abrupt changes in aircraft control variables at the waypoints. Because such abrupt control variations are not desirable, smooth trajectory segments will need to be introduced in the vicinity of the waypoints to ensure practical implementability. Note that these trajectory segments can introduce uncertainties in the aircraft position near the waypoints. It is assumed that these smooth trajectory segments can be introduced without adversely affecting the conflict resolution solutions.

As an aside, the cubic spline parameterization produces smooth trajectories as well as smooth control settings. However, it requires more computation time and tends to introduce anomalies in the path by virtue of its fitting a fixed-order, smooth curve through a given set of points. Because of these factors, although a few cubic spline parameterization cases were studied during this research, the emphasis of the present paper is on the use of a piecewise-linear trajectory parameterization.

C. Conflict Envelope Definition

According to the currently accepted definition,[14] a conflict is said to occur if the altitude difference between an aircraft pair occupying the same down-range and cross-range coordinates is less than 2000 ft or if the two aircraft are closer than 5 n mile while flying at the same altitude. It is convenient to conceptualize these requirements by imagining that each aircraft is centered inside a hypothetical conflict box with dimensions of 10-n mile length and breadth, with 4000 ft thickness. Any other aircraft entering this hypothetical box can be considered as causing a conflict.

Although this definition of a conflict is simple to conceptualize, it can cause severe numerical difficulties due to the corners present in such a conflict envelope. For instance, in certain kinematic conflict configurations, small motions of the aircraft can cause a large change in the conflict status. Such large changes can cause numerical instabilities in the conflict resolution algorithm. Alternate definitions can be developed to enforce the aircraft separation constraint without introducing such numerical discontinuities. Possibilities include quadratic surfaces[31] and superquadric surfaces.[32,33] An oblate spheroidal conflict envelope is an example of a quadratic surface. The oblate spheroidal conflict envelope has an elliptical cross section in the vertical plane and a circular cross-section in the horizontal plane. To approximate the FAA definition of the conflict envelope, the semimajor axis of the elliptical cross section can be chosen as 5 n mile, and the semiminor axis can be set at 2000 ft. The interaircraft distance is the distance from an aircraft's c.g. to another aircraft's conflict envelope. Using this geometry, the distance $r_{i,j}$ between any two aircraft i and j can be defined using the expression

$$r_{i,j} = \sqrt{\Delta x_{i,j}^2 + \Delta y_{i,j}^2 + \Delta h_{i,j}^2} - \sqrt{\frac{a^2 b^2\left(\Delta x_{i,j}^2 + \Delta y_{i,j}^2 + \Delta h_{i,j}^2\right)}{a^2 \Delta h_{i,j}^2 + b^2\left(\Delta x_{i,j}^2 + \Delta y_{i,j}^2\right)}} \qquad i \neq j \quad (9)$$

where a is the conflict envelope semimajor axis, b is the semiminor axis, and $\Delta x_{i,j}$, $\Delta y_{i,j}$, and $\Delta h_{i,j}$ are the components of the relative position vector between any two aircraft i and j. It can be verified that, if $a = b$, the conflict envelope degenerates into a sphere. A conflict can be defined as a situation in which any of the interaircraft distances $r_{i,j}$ falls below zero. Note that if n aircraft are involved in an air traffic control situation, a maximum of $n(n-1)/2$ conflicts can simultaneously arise. The resulting interaircraft distances can be represented as a symmetric matrix with zeros along the diagonal. Any conflict resolution methodology must ensure that all of the off-diagonal elements of the interaircraft distance matrix remain above zero at all times. A further refinement of the interaircraft conflict definition is to introduce a matrix norm in the formulation. The Frobenius norm or the matrix 2-norm[34] is a candidate measure that can be used to express the interaircraft conflict in a compact form. These more advanced formulations will be pursued during future research.

III. Direct Iterative Methods for Conflict Resolution

The conflict resolution process consists of modifying the aircraft nominal airspeed, altitude, and down-range and cross range trajectory components to ensure that every aircraft stays out of the every other aircraft's conflict envelope while executing their flight plans. These trajectory perturbations must not significantly change the arrival times at the specified terminal point and should not result in excessive maneuvering. Moreover, conflict resolution trajectory changes should be such that they do not lead to future conflicts. In some cases, it may be desirable to include fuel conservative measures in the conflict resolution performance objectives. Because the atmospheric perturbations and piloting techniques introduce uncertainties in the aircraft trajectories, the trajectory synthesis should be such that the worst-case perturbations in each aircraft trajectory still preserves the integrity of the conflict resolution process.

From the foregoing requirements, it can be observed that the conflict resolution problem can be formulated as a multiparticipant trajectory optimization problem. This problem can be cast in several different forms by assigning roles to individual aircraft. For instance, it can be treated as a single objective trajectory optimization problem in which every aircraft performs conflict resolution maneuvers that optimize a defined performance index. Alternatively, the conflict resolution problem can be cast as a series of one-sided optimal control problems by considering a pair of aircraft at a time, with one aircraft operating along the nominal trajectory and the other aircraft making the necessary trajectory adjustments required for conflict resolution. Finally, it can be treated as a multiobjective optimization problem in which each participant makes the least amount of trajectory deviations necessary for maintaining the interaircraft distance above a minimum specified value while simultaneously optimizing its performance criterion. Such approaches to the conflict resolution problem can be considered as a cooperative conflict resolution approach. Other variations of the conflict resolution problem formulation are presented in Sec. IV.

All of these approaches require the definition of performance indices that reflect the operational requirements of the conflict resolution problem. Performance indices considered in the present research are linear combinations of flight time and fuel and integral square of the deviations from the nominal flight paths. The constraints in the problem include the minimum permissible interaircraft separation and aircraft performance limits. Additionally, aircraft trajectories may be required to pass through a series of metering waypoints, as is required in the current air traffic control environment. The resulting problem is a state-constrained optimal control problem[18,19] that can be approached using well-developed mathematical techniques.

The necessary conditions for the optimality of state-constrained optimal control problems can result in multipoint boundary-value problems.[18] There are two major families of techniques useful for solving these problems. First, the conflict resolution problem can be converted into a parameter optimization problem by parameterizing the trajectories. Trajectory parameterization methods described in the Sec. II are useful for this purpose. The resulting optimization methods are commonly known as direct methods. The second family of solution approaches consists of satisfying the necessary conditions for optimality by iteratively solving simplified versions of the multipoint boundary-value problem. This latter family of approaches is termed as indirect methods in the literature. Indirect methods are especially suitable for the derivation of feedback guidance laws. Solutions to the conflict resolution problem via trajectory parameterization will be discussed in this section. Conflict resolution guidance law derivation using indirect methods will be considered in Sec. IV.

Two versions of the direct conflict resolution technique are discussed in the following subsections. In Sec. III.A, the conflict resolution problem is formulated as a single-objective optimization problem. The sequential quadratic programming (SQP) method[35] is used to solve the resulting optimization problem. The conflict resolution problem is formulated and solved as a multiobjective optimization problem in Sec. III.B. The goal attainment method[26,27] is used to obtain the solutions to the multiobjective optimization problem.

The computational techniques are discussed in Secs. III.A and III.B, followed by a discussion of the numerical results in Sec. III.C. Solutions are obtained for conflict situations involving two, three, four, and six aircraft. All of the conflict resolution solutions are obtained using piecewise-linear trajectory parameterization. Comparisons between the single-objective conflict resolution solutions and multiobjective solutions are also given in Sec. III.C.

A. Conflict Resolution as a Single Objective Optimization Problem

Starting from a set of nominal waypoint sequences, the objective is to synthesize trajectories that resolve conflicts while optimizing the performance index. The performance index is the sum of the integral deviations of each aircraft from its nominal trajectory. Inequality constraints are included for interaircraft distance, maximum and minimum airspeed, maximum climb and descent rates, maximum and minimum altitudes, maximum roll angle, maximum and minimum load factors, and maximum and minimum thrust magnitudes. The nominal trajectories of each aircraft are specified in terms of a set of waypoints (x, y, h, t). The present formulation of the trajectory optimization problem allows the waypoints to be constrained in any desired manner. Physical characteristics of each aircraft such as mass, drag coefficient, and fuel consumption rate also are included in the study. Another criterion used was the sum of individual aircraft flight times and fuel consumptions, but results are not included for this case.

The algebraic transformations that relate thrust, load factor, and roll angle to the acceleration components discussed in Sec. II are included in the formulation so as to permit the imposition of aircraft performance constraints without including the aircraft equations of motion. Trajectories are parameterized using the piecewise-linear parameterization scheme discussed in Sec. II. The optimization algorithm adjusts the trajectory parameters to resolve any existing conflicts while minimizing cost and meeting the aircraft performance constraints. The optimization problem is currently set up to accommodate a maximum of 10 aircraft, although this can easily be increased if necessary.

Optimal trajectories are computed using the SQP method. The SQP method has been found highly effective in solving a wide variety of constrained nonlinear programming problems.[35] The SQP method solves optimization problems of the form

$$\begin{aligned} &\text{minimize } f(\boldsymbol{x}) \\ &\text{subject to } \boldsymbol{g}(\boldsymbol{x}) \leq 0 \end{aligned} \tag{10}$$

by approximating the performance index $f(\boldsymbol{x})$ by a quadratic function and the constraints $\boldsymbol{g}(\boldsymbol{x})$ by linear functions. The SQP method attempts to imitate the Newton's method, which is known to converge to the optimal solution in the least number of steps for quadratic problems.[35] In expression (10) $\boldsymbol{x}$ is an n vector of optimization parameters. A more detailed description of the SQP method can be found in Refs. 35–37. The Optimization Toolbox of the MATLAB™ software[37] is used in the present investigation for implementing the sequential quadratic programming algorithm.

An interesting aspect of the single-objective conflict resolution method using the SQP algorithm is that the sensitivity of the cost function and constraints with respect to individual trajectory parameters can be readily computed using the gradient of the cost function, the Hessian matrix, and the Jacobian matrix of the constraints. These sensitivities can be then used by individual aircraft to further negotiate changes to their path. For instance, large-trajectory changes can be made in regions of low-cost/constraint sensitivity, whereas trajectory perturbations must be limited in the high-sensitivity regions. Such information can be valuable for the practical implementation of the conflict resolution method.

Numerical results using the single-objective formulation of the conflict resolution problem will be given in Sec. III.C. An alternate formulation of the conflict resolution problem is discussed in the following section.

B. Conflict Resolution as a Multiple-Objective Optimization Problem

The single-objective formulation of the conflict resolution problem is difficult to justify in the free-flight environment because individual aircraft involved in the maneuvers may not be interested in optimizing a single performance index. For instance, some of the aircraft may place larger emphasis on flight time than fuel, whereas other aircraft may be interested in minimizing fuel consumption. Multiple-objective optimization methods allow the use of more than one cost function and are perhaps more realistic in representing the true nature of the multiaircraft conflict resolution problem. Note that the multiple-objective formulation automatically allows the simultaneous inclusion of cooperating and competing objectives if any are present in the problem.

The cost function in the multiobjective conflict resolution problem is a vector. Several algorithms have been reported in the literature for the solution of multiple-objective optimization problems.[26] Multiple-objective optimization problems seek a set of noninferior points as their solution. Noninferiority implies that any further improvement in one objective would result in a degradation in another objective. Thus, all points in the solution set minimize the performance objectives to the maximum possible extent. The technique selected for use in the present research is the goal attainment method discussed in Refs. 26, 27, and 37. Here, the objectives and the constraints are all represented as goals to be satisfied, and the degree to which these goals are met is adjusted using weighting functions. The goal attainment formulation allows the imposition of both inequality and equality constraints.

Mathematically, the goal attainment method can be stated as

$$\text{minimize } \gamma, \quad \text{where} \quad \boldsymbol{f}(\boldsymbol{x}) - \boldsymbol{w}\gamma \leq \boldsymbol{f}^* \tag{11}$$

Note that $\boldsymbol{f}(\boldsymbol{x})$ is a vector that includes the cost functions and the constraints. The goals are contained in the vector $\boldsymbol{f}^*$, and the weights are in the vector $\boldsymbol{w}$. The scalar γ is a measure of how far away a solution point is from the goals. Thus, the term $\boldsymbol{w}\gamma$ can be considered as the slackness in the problem. Individual weights capture the fact that in many design problems, a goal need not be met exactly and one is more interested in arriving at an optimal tradeoff between conflicting objectives. The weighting vector defines the direction from the goal point to the feasible region. As γ is varied, the feasible region changes size, until the constraint boundaries converge to a solution point. One of the advantages of the goal attainment formulation is that it can be solved as an SQP problem. As in the case of single-objective conflict resolution discussed in the preceding section, the Optimization Toolbox of MATLAB is used to implement the goal attainment method.

Conflict resolution trajectories involving two, three, four, and six aircraft have been generated using the goal attainment method. Some of these will be illustrated in Sec. III.C.

C. Conflict Resolution Results

The conflict resolution methods developed in Secs. III.A and III.B are evaluated in a sample conflict scenario in this section. The results for single-objective and multiple-objective formulations corresponding a six-aircraft conflict will be given here. Models are based on data from Refs. 28 and 38.

This conflict scenario is generated by including six merging aircraft, five of them flying at 30,000 ft and the sixth aircraft merging from 32,000 ft altitude. Such a conflict can arise during merging operations near an air traffic control center. The initial aircraft speeds are: $V_1 = 780$ ft/s, $V_2 = 733$ ft/s, $V_3 = 548$ ft/s, $V_4 = 708$ ft/s,

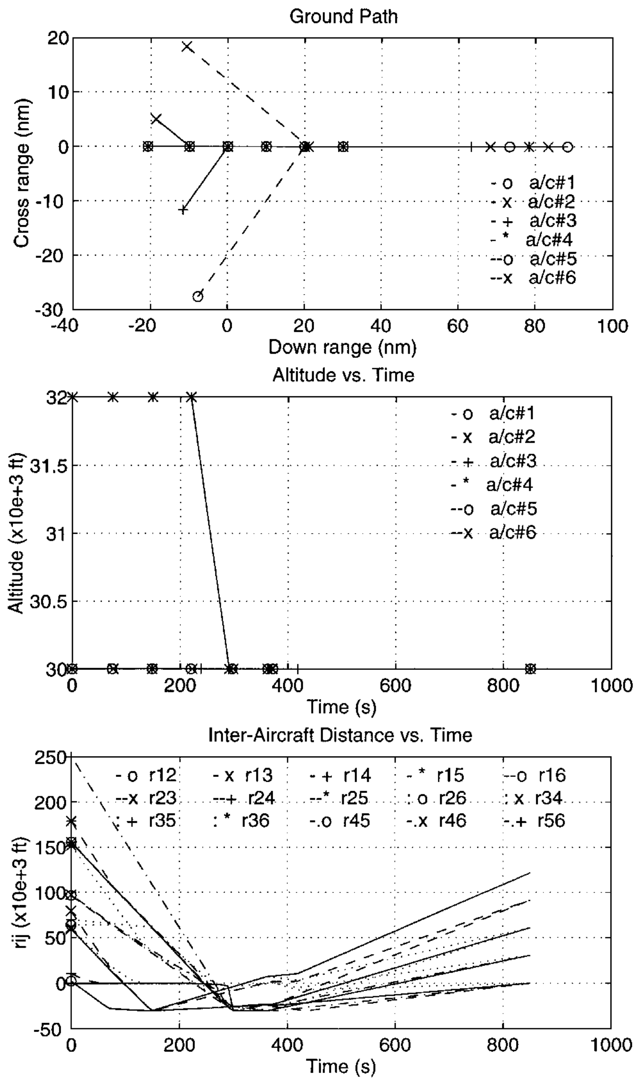

Fig. 2 Nominal trajectories for six-aircraft conflict resolution.

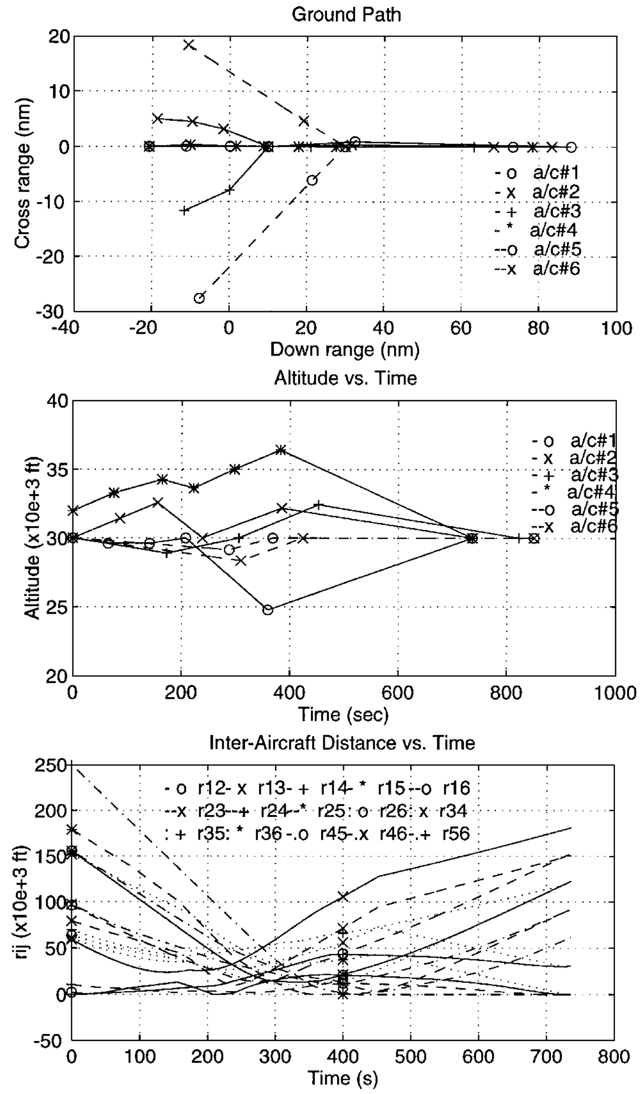

Fig. 3 Single-objective conflict resolution for the six-aircraft geometry.

$V_5 = 586$ ft/s, and $V_6 = 490$ ft/s. The aircraft parameters used in this conflict scenario are: mass = 185,000 lbm, drag coefficient $C_D = 0.05$, and wing area $S = 1700$ ft^2.

Fuel consumption rate is assumed be a quadratic function of airspeed, with the constant of proportionality being 0.3×10^{-6} lbm-s/ft^2. The performance constraints imposed on the conflict resolution problem are: 340 ft/s < airspeed < 900 ft/s; 0.9 g < load factor < 1.1 g; 1000 lbf < thrust < 48,000 lbf; 20,000 ft < altitude < 40,000 ft; and −3000 ft/min < rate of climb < 1000 ft/min.

The arrival order at the end of the maneuvers is assumed to be specified. Nominal trajectories for the aircraft are given in Fig. 2. It may be observed from the interaircraft distance plot that the conflicts between aircraft occur right from the beginning, and by 300 s, every aircraft is in conflict with the rest. Note that the conflict resolution strategies are far from obvious in this situation.

Conflict resolution using the single-objective formulation is given in Fig. 3. The sum of the squares of the deviations from the nominal trajectories is used as the performance index in these optimization runs. Note that the methodology has modified each aircraft trajectory sufficiently to completely resolve the conflicts. Trajectory corrections have been introduced in both the vertical plane and the horizontal plane. Conflict resolution using the multiple-objective formulation is shown in Fig. 4. Although it is rather difficult to gauge the differences between the two approaches, an analysis of the sensitivities of the trajectories with respect to the constraints shows that the multiobjective formulation results in a more robust solution.

One problem with these results is that some aircraft are required to make significant changes in altitude or several climbs and descents which, for practical reasons, are undesirable. There are several possible ways to control this behavior: tighten the constraints on rate of climb, introduce more constraints on acceleration, or perhaps simply scale the altitude to make it more influential on the cost function. Note, though, that the theoretically optimal maneuver involves altitude changes, even with fuel consumption included in the cost.

It should be noted that there is no guarantee that a solution exists with these methods, although the methods never failed in any of the many cases tested. Because the problem is nonlinear and a gradient-based search is employed for finding optimal trajectories, it is impossible to ascertain the nature of the minimum obtained. As a practical matter, however, the question of local optimality vs global optimality has relatively minor significance, as long as the converged solution is conflict free.

IV. Closed-Loop Guidance Law for Conflict Resolution

The preceding section discussed iterative techniques for conflict resolution. The conflict resolution problem is formulated as a closed-loop guidance problem in this section. A guidance law is developed based on the assumption that the nominal paths are the desirable paths and that any perturbation from these paths should only be made to achieve conflict resolution. The guidance law derivation for two aircraft will be illustrated. Generalization to multiple aircraft conflicts is a future research item. From a guidance law derivation point of view, the aircraft involved in a potential conflict can act in the following three fashions.

1) All aircraft involved in the potential conflict cooperate with each other for resolving the conflict. This would be the case when

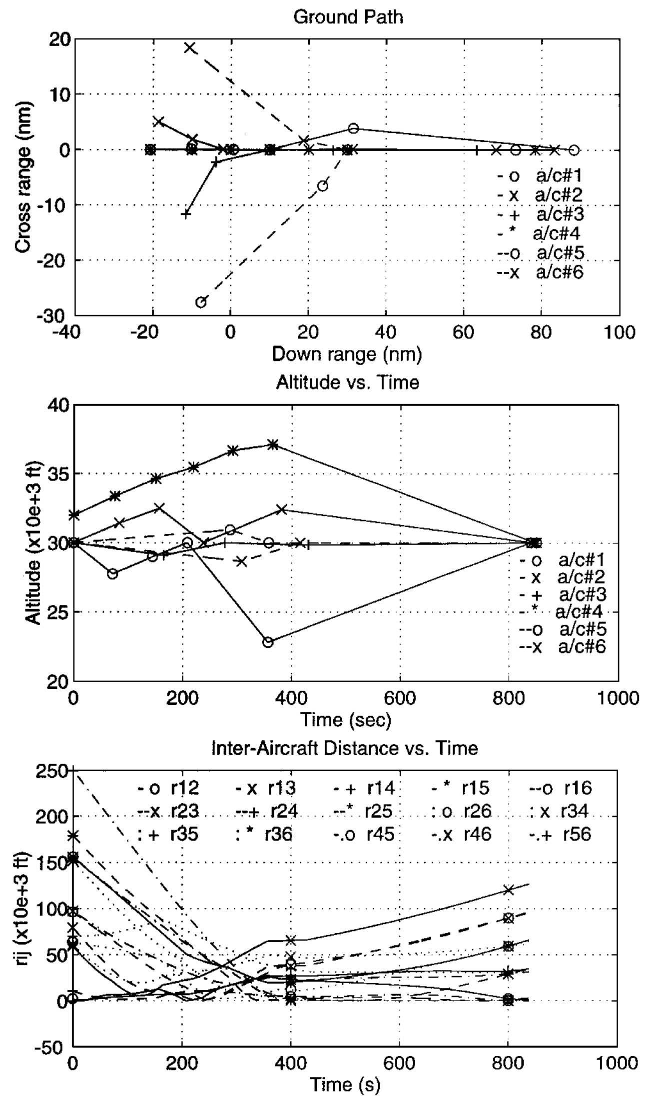

Fig. 4 Multiple-objective conflict resolution for the six-aircraft geometry.

every aircraft is aware of every other aircraft in the vicinity, no aircraft has any priority over other aircraft, and all aircraft have comparable maneuverability. The resulting conflict resolution process will be termed here as cooperative conflict resolution.

2) Some aircraft involved in the potential conflict continue to fly nominal paths, while others take on the responsibility for resolving the conflicts. This situation can arise because of three distinct reasons. First, some of the aircraft may be significantly be more agile than other aircraft. In this case, the agile aircraft may be able to resolve conflicts much faster and more efficiently than nonagile aircraft. The second mechanism for such a scenario is when the aircraft are assigned a priority ordering, such that certain aircraft are allowed to stay on their nominal trajectories, while the others are made responsible for conflict resolution. The third situation where this might occur is when some of the aircraft are not aware of other aircraft in the vicinity and, hence, continue on their nominal paths, while the remaining aircraft have to take responsibility for conflict resolution. These scenarios can be termed as noncooperative conflict resolution.

3) The third operational mode corresponds to the case of some of the aircraft in the airspace maneuver in such a way as to increase the potential for conflict. This situation may arise whenever some of the aircraft are not aware of other aircraft in the vicinity and may maneuver in such a way as to increase the conflicts. Alternatively, to guarantee conflict resolution under the worst case, each properly equipped aircraft in the vicinity may decide to maneuver under the assumption that the unequipped aircraft in the vicinity may alter their trajectories in such a way as to cause conflicts. This case will be termed here as adversarial or competitive conflict resolution. Note this conflict scenario has a solution only if the conflict-resolving aircraft have superior data gathering and maneuvering capabilities when compared with the apparently adversarial aircraft.

Characterizing the conflict resolution problem in terms of these categories serves to capture the degree of optimism of the aircraft involved in a given conflict scenario. Thus, the first mode corresponds to a fully optimistic conflict resolution process, whereas the second is more neutral. The third formalism is pessimistic and will lead to highly conservative conflict resolution maneuvers.

Guidance law development will assume that each aircraft in the airspace has selected a nominal waypoint sequence based on some other criteria. The criteria for nominal trajectory selection may include the minimization of flight time and fuel consumption to reach a selected set of final conditions while satisfying aircraft and environmental constraints. They may also be based on prevailing atmospheric conditions and the need for coordinating operations with other aircraft in the air transportation system. Conflict resolution guidance law introduces perturbations on these nominal waypoints to resolve the conflicts. Conflict resolution perturbations can be introduced in several different ways. For instance, the trajectory perturbations can be based on a set of rules capturing the maneuvering strategies in every given conflict situation. Alternatively, one may cast the trajectory perturbation problem as an optimal control problem[18] that minimizes the deviations from the nominal trajectories while achieving conflict resolution. In this framework, penalties can be placed on the magnitudes of acceleration, velocity, and position components. This approach is followed in the present research.

To formulate the conflict resolution problem in this manner, perturbations in each aircraft trajectory are first expressed as

$$\delta x_1 = x_1 - \bar{x}_1, \qquad \delta y_1 = y_1 - \bar{y}_1, \qquad \delta z_1 = z_1 - \bar{z}_1 \tag{12}$$

$$\delta x_2 = x_2 - \bar{x}_2, \qquad \delta y_2 = y_2 - \bar{y}_2, \qquad \delta z_2 = z_2 - \bar{z}_2 \tag{13}$$

where an overbar denotes the nominal value. Differentiating the expressions (12) and (13) twice with respect to time leads to the perturbed equations of motion

$$\begin{aligned} \delta\ddot{x}_1 &= \ddot{x}_1 - \ddot{\bar{x}}_1 = u_1, & \delta\ddot{x}_2 &= \ddot{x}_2 - \ddot{\bar{x}}_2 = u_2 \\ \delta\ddot{y}_1 &= \ddot{y}_1 - \ddot{\bar{y}}_1 = v_1, & \delta\ddot{y}_2 &= \ddot{y}_2 - \ddot{\bar{y}}_2 = v_2 \\ \delta\ddot{z}_1 &= \ddot{z}_1 - \ddot{\bar{z}}_1 = w_1, & \delta\ddot{z}_2 &= \ddot{z}_2 - \ddot{\bar{z}}_2 = w_2 \end{aligned} \tag{14}$$

where u_1, u_2, v_1, v_2, w_1, and w_2 are the perturbations in the aircraft acceleration components along the down-range, cross-range, and vertical directions. These quantities can be related to perturbations in actual control variables of the aircraft using the expressions derived in Sec. II. The reference values of the velocity and position components along the nominal trajectory can be determined using the given waypoint sequence.

Deviations from nominal trajectory can be characterized in terms of the Euclidean norm of the velocity and position perturbations. The position perturbations are not included in the performance index used in the present research. Thus, a performance index of the form

$$\begin{aligned} \min_{u_1,u_2,v_1,v_2,w_1,w_2} \frac{1}{2}\int_0^{t_f} \Bigg[& \alpha_1^2\left(\delta\dot{x}_1^2 + \delta\dot{y}_1^2 + \delta\dot{z}_1^2\right) \\ & + \alpha_2^2\left(\delta\dot{x}_2^2 + \delta\dot{y}_2^2 + \delta\dot{z}_2^2\right) + \frac{1}{\beta_1^2}\left(u_1^2 + v_1^2 + w_1^2\right) \\ & + \frac{1}{\beta_2^2}\left(u_2^2 + v_2^2 + w_2^2\right)\Bigg] \mathrm{d}t \end{aligned} \tag{15}$$

is used for the conflict resolution problem. Weighting factors α_1, β_1, α_2, and β_2 can be used to adjust the relative importance of velocity and acceleration perturbations. They can also be used to cast the problem as a cooperative, noncooperative, or adversarial conflict resolution problem. For instance, using equal weights for both aircraft will result in cooperative conflict resolution, whereas using very large weights for one of the aircraft will produce the noncooperative conflict resolution solution. Changing the signs of the terms for one of the aircraft in the performance index will produce the adversarial conflict resolution solution. For the sake of illuminating the guidance law development, the present research will employ equal weighting factors for both aircraft.

Because the conflict resolution perturbations must begin and end on the nominal trajectories, they must satisfy the boundary conditions

$$\delta x_1(t_0) = \delta\dot{x}_1(t_0) = \delta x_2(t_0) = \delta\dot{x}_2(t_0) = 0$$
$$\delta x_1(t_f) = \delta\dot{x}_1(t_f) = \delta x_2(t_f) = \delta\dot{x}_2(t_f) = 0 \tag{16}$$

The minimum interaircraft distance requirement between the two aircraft at the time of conflict can be expressed as an interior-point equality constraint of the form

$$(\bar{x}_1 + \delta x_1 - \bar{x}_2 - \delta x_2)^2 + (\bar{y}_1 + \delta y_1 - \bar{y}_2 - \delta y_2)^2 + (\bar{z}_1 + \delta z_1 - \bar{z}_2 - \delta z_2)^2 - (\bar{x}_1 - \bar{x}_2)^2 - (\bar{y}_1 - \bar{y}_2)^2 - (\bar{z}_1 - \bar{z}_2)^2|_{t=t_c} = R^2_{\min} \tag{17}$$

or

$$(\delta x_1 - \delta x_2)^2 + (\delta y_1 - \delta y_2)^2 + (\delta z_1 - \delta z_2)^2 + 2(\bar{x}_1 - \bar{x}_2)(\delta x_1 - \delta x_2) + 2(\bar{y}_1 - \bar{y}_2)(\delta y_1 - \delta y_2) + 2(\bar{z}_1 - \bar{z}_2)(\delta z_1 - \delta z_2)|_{t=t_c} = R^2_{\min} \tag{18}$$

A conflict is assumed to occur at time t_c, and $R_{\min}$ is the desired additional separation distance between the two aircraft at the point of closest approach. It is assumed that $0 \le t_c \le t_f$. In the case of n conflicting aircraft, the optimal trajectories will have to satisfy $n(n-1)/2$ constraints of this form.

Additionally, to ensure that the separation between the aircraft does not decrease after t_c, the relative velocity along the direction of minimum separation will be required to be zero, giving the additional interior-point constraint

$$(\delta x_1 - \delta x_2)(\delta\dot{x}_1 - \delta\dot{x}_2) + (\delta y_1 - \delta y_2)(\delta\dot{y}_1 - \delta\dot{y}_2) + (\delta z_1 - \delta z_2)(\delta\dot{z}_1 - \delta\dot{z}_2) + (\dot{\bar{x}}_1 - \dot{\bar{x}}_2)(\delta x_1 - \delta x_2) + (\bar{x}_1 - \bar{x}_2)(\delta\dot{x}_1 - \delta\dot{x}_2) + (\dot{\bar{y}}_1 - \dot{\bar{y}}_2)(\delta y_1 - \delta y_2) + (\bar{y}_1 - \bar{y}_2)(\delta\dot{y}_1 - \delta\dot{y}_2) + (\dot{\bar{z}}_1 - \dot{\bar{z}}_2)(\delta z_1 - \delta z_2) + (\bar{z}_1 - \bar{z}_2)(\delta\dot{z}_1 - \delta\dot{z}_2)|_{t=t_c} = 0 \tag{19}$$

In the interest of maintaining simplicity, for this initial study, the conflict envelope has been assumed to spherical. The guidance law derivation along one of the coordinate axes will be illustrated in the following.

A. Conflict Resolution Guidance Law in the Down-Range Direction

To limit the complexity of the derivations in this initial research, it will be assumed that the conflict resolution problem can be solved independently along each coordinate direction by generating the identical amounts of separation along each axis. For instance, a performance index of the form

$$\min_{u_1,u_2} \frac{1}{2}\int_0^{t_f}\left[\left(\delta\dot{x}_1^2 + \delta\dot{x}_2^2\right) + \frac{1}{\beta^2}\left(u_1^2 + u_2^2\right)\right]\mathrm{d}t \tag{20}$$

is used in the down-range direction. Similar performance indices can be used along the cross-range and altitude directions. The interior-point equality constraints are then expressed as

$$\left[(\delta x_1 - \delta x_2)^2 + 2(\bar{x}_1 - \bar{x}_2)(\delta x_1 - \delta x_2)\right]\Big|_{t=t_c} = r^2_{\min} \tag{21}$$

$$\left[(\delta x_1 - \delta x_2)(\delta\dot{x}_1 - \delta\dot{x}_2) + (\dot{\bar{x}}_1 - \dot{\bar{x}}_2)(\delta x_1 - \delta x_2) + (\bar{x}_1 - \bar{x}_2)(\delta\dot{x}_1 - \delta\dot{x}_2)\right]\Big|_{t=t_c} = 0 \tag{22}$$

where $r_{\min}$ is the additional separation that needs to be generated along the down-range direction. Note that the second interior-point constraint further simplifies to $(\delta\dot{x}_1 - \delta\dot{x}_2) = 0$ if $(\dot{\bar{x}}_1 - \dot{\bar{x}}_2) = 0$ at $t = t_c$.

Optimal control problems with interior-point constraints can be treated as two distinct optimal control problems. The first one is defined in the interval $[0, t_c]$ with the interior-point constraints as the terminal conditions, whereas the second optimal control problem is defined in the interval $[t_c, t_f]$ with interior point constraints as the initial conditions.

The optimal control problem is solved as follows. Using the performance index (20), define the variational Hamiltonian[18] as

$$H = \tfrac{1}{2}\left[\left(\delta\dot{x}_1^2 + \delta\dot{x}_2^2\right) + (1/\beta^2)\left(u_1^2 + u_2^2\right)\right] + \lambda_1 u_1 + \lambda_2 u_2 + \lambda_3\delta\dot{x}_1 + \lambda_4\delta\dot{x}_2 \tag{23}$$

The Euler–Lagrange equations are of the form

$$\dot{\lambda}_1 = -\delta\dot{x}_1 - \lambda_3, \qquad \dot{\lambda}_3 = 0, \qquad u_1 = -\beta^2\lambda_1 \tag{24}$$

$$\dot{\lambda}_2 = -\delta\dot{x}_2 - \lambda_4, \qquad \dot{\lambda}_4 = 0, \qquad u_2 = -\beta^2\lambda_2 \tag{25}$$

The first two sets of costate equations and optimality conditions can be solved in closed-form as

$$\lambda_1 = a_1 e^{\beta t} + b_1 e^{-\beta t}, \qquad u_1 = -\beta^2\left(a_1 e^{\beta t} + b_1 e^{-\beta t}\right) \tag{26}$$

$$\lambda_2 = a_2 e^{\beta t} + b_2 e^{-\beta t}, \qquad u_2 = -\beta^2\left(a_2 e^{\beta t} + b_2 e^{-\beta t}\right) \tag{27}$$

where a_1, b_1, a_2, and b_2 are the arbitrary constants in the solution. The expressions for optimal control are next substituted in the equations of motion (14) and integrated to yield

$$\delta\dot{x}_1 = c_1 - \beta\left(a_1 e^{\beta t} - b_1 e^{-\beta t}\right)$$
$$\delta\dot{x}_2 = c_2 - \beta\left(a_2 e^{\beta t} - b_2 e^{-\beta t}\right) \tag{28}$$

$$\delta x_1 = d_1 + c_1 t - \left(a_1 e^{\beta t} + b_1 e^{-\beta t}\right)$$
$$\delta x_2 = d_2 + c_2 t - \left(a_2 e^{\beta t} + b_2 e^{-\beta t}\right) \tag{29}$$

where c_1, c_2, d_1, and d_2 are arbitrary constants in the solution. Using these expressions in Eqs. (24) and (25), the costates λ_3 and λ_4 can be found to be

$$\lambda_3 = -c_1, \qquad \lambda_4 = -c_2 \tag{30}$$

The arbitrary constants a_1, b_1, c_1, d_1, a_2, b_2, c_2, and d_2 can be evaluated using the given boundary conditions.

The solution on the interval $[t_c, t_f]$ will be discussed first because it is easier to solve. Applying the boundary conditions at t_c and t_f yields the expressions

$$a_1\left(1 - e^{\beta(t_f - t_c)}\right) - b_1\left(1 - e^{-\beta(t_f - t_c)}\right) = \frac{\delta\dot{x}_1(t_f) - \delta\dot{x}_1(t_c)}{\beta} \tag{31}$$

$$a_2\left(1 - e^{\beta(t_f - t_c)}\right) - b_2\left(1 - e^{-\beta(t_f - t_c)}\right) = \frac{\delta\dot{x}_2(t_f) - \delta\dot{x}_2(t_c)}{\beta} \tag{32}$$

$$a_1\left[1 + \beta(t_f - t_c) - e^{\beta(t_f - t_c)}\right] + b_1\left[1 - \beta(t_f - t_c) - e^{-\beta(t_f - t_c)}\right] = \delta x_1(t_f) - \delta x_1(t_c) - [t_f - t_c]\delta\dot{x}_1(t_c) \tag{33}$$

$$a_2\left[1 + \beta(t_f - t_c) - e^{\beta(t_f - t_c)}\right] + b_2\left[1 - \beta(t_f - t_c) - e^{-\beta(t_f - t_c)}\right] = \delta x_2(t_f) - \delta x_2(t_c) - [t_f - t_c]\delta\dot{x}_2(t_c) \tag{34}$$

$$c_1 = \delta\dot{x}_1(t_c) + \beta(a_1 - b_1), \qquad d_1 = \delta x_1(t_c) + (a_1 + b_1) \tag{35}$$

$$c_2 = \delta\dot{x}_2(t_c) + \beta(a_2 - b_2), \qquad d_2 = \delta x_2(t_c) + (a_2 + b_2) \tag{36}$$

These expressions can be solved to obtain the trajectory perturbations required to restore the aircraft to the nominal trajectories after conflict resolution.

Now the solution on the interval $[0, t_c]$ will be derived. To handle the state-constrained optimal control problem between the initial time and the point of closest approach, define a final-time constraint, using Eqs. (21) and (22), as

$$\Phi(t_c) = \mu_1\left[(\delta x_1 - \delta x_2)^2 + 2(\bar{x}_1 - \bar{x}_2)(\delta x_1 - \delta x_2)|_{t=t_c} - r_{\min}^2\right] + \mu_2\left[(\delta x_1 - \delta x_2)(\delta\dot{x}_1 - \delta\dot{x}_2) + (\dot{\bar{x}}_1 - \dot{\bar{x}}_2)(\delta x_1 - \delta x_2) + (\bar{x}_1 - \bar{x}_2)(\delta\dot{x}_1 - \delta\dot{x}_2)|_{t=t_c}\right] \quad (37)$$

where μ_1 and μ_2 are two undetermined constants. Condition (22) imposes constraints on the value of the costates at $t = t_c$ as

$$\lambda_1(t_c) = \mu_2\left[(\delta x_1 - \delta x_2) + (\bar{x}_1 - \bar{x}_2)|_{t=t_c}\right] \quad (38)$$

$$\lambda_2(t_c) = -\mu_2\left[(\delta x_1 - \delta x_2) + (\bar{x}_1 - \bar{x}_2)|_{t=t_c}\right] \quad (39)$$

implying $\lambda_1(t_c) = -\lambda_2(t_c)$. From Eqs. (26) and (27), this leads to the result

$$(a_1 + a_2)e^{\beta t_c} = -(b_1 + b_2)e^{-\beta t_c} \quad (40)$$

Similarly,

$$\lambda_3(t_c) = \mu_1\left[2(\delta x_1 - \delta x_2) + 2(\bar{x}_1 - \bar{x}_2)|_{t=t_c}\right] + \mu_2\left[(\delta\dot{x}_1 - \delta\dot{x}_2) + (\dot{\bar{x}}_1 - \dot{\bar{x}}_2)|_{t=t_c}\right] \quad (41)$$

$$\lambda_4(t_c) = -\mu_1\left[2(\delta x_1 - \delta x_2) + 2(\bar{x}_1 - \bar{x}_2)|_{t=t_c}\right] - \mu_2\left[(\delta\dot{x}_1 - \delta\dot{x}_2) + (\dot{\bar{x}}_1 - \dot{\bar{x}}_2)|_{t=t_c}\right] \quad (42)$$

which implies $\lambda_3(t_c) = -\lambda_4(t_c)$. This relationship, together with Eq. (30), gives the values of c_1 and c_2. Substituting for these quantities in Eq. (28) with $t = 0$ gives

$$(a_1 + a_2) - (b_1 + b_2) = \frac{-\delta\dot{x}_2(0) - \delta\dot{x}_1(0)}{\beta} \quad (43)$$

Equations (43) and (40) can be used to obtain

$$(b_1 + b_2) = \frac{\delta\dot{x}_2(0) + \delta\dot{x}_1(0)}{\beta(1 + e^{-2\beta t_c})}, \qquad (a_1 + a_2) = -\left[\frac{\delta\dot{x}_2(0) + \delta\dot{x}_1(0)}{\beta(1 + e^{-2\beta t_c})}\right]e^{-2\beta t_c} \quad (44)$$

The terminal constraints yield the following relationships:

$$(\delta x_1 - \delta x_2)|_{t=t_c} = -(\bar{x}_1 - \bar{x}_2) \pm \sqrt{(\bar{x}_1 - \bar{x}_2)^2 + r_{\min}^2}\Big|_{t=t_c} \quad (45)$$

$$(\delta\dot{x}_1 - \delta\dot{x}_2)|_{t=t_c} = \left.\frac{-(\dot{\bar{x}}_1 - \dot{\bar{x}}_2)\left[-(\bar{x}_1 - \bar{x}_2) \pm \sqrt{(\bar{x}_1 - \bar{x}_2)^2 + r_{\min}^2}\right]}{\pm\sqrt{(\bar{x}_1 - \bar{x}_2) + r_{\min}^2}}\right|_{t=t_c} \quad (46)$$

From the closed-form solutions in Eqs. (28) and (29),

$$(\delta\dot{x}_1 - \delta\dot{x}_2)|_{t-t_c} = c_1 - c_2 - \beta(a_1 - a_2)e^{\beta t_c} + \beta(b_1 - b_2)e^{-\beta t_c} \quad (47)$$

$$(\delta x_1 - \delta x_2)|_{t=t_c} = d_1 - d_2 + (c_1 - c_2)t_c - (a_1 - a_2)e^{\beta t_c} - (b_1 - b_2)e^{-\beta t_c} \quad (48)$$

Next, substituting for the arbitrary constants in terms of the boundary conditions,

$$(\delta\dot{x}_1 - \delta\dot{x}_2)|_{t-t_c} = [\delta\dot{x}_1(0) - \delta\dot{x}_2(0)] + (a_1 - a_2)\beta[1 - e^{\beta t_c}] - (b_1 - b_2)\beta[1 - e^{-\beta t_c}] \quad (49)$$

$$(\delta x_1 - \delta x_2)|_{t=t_c} = [\delta x_1(0) - \delta x_2(0)] + [\delta\dot{x}_1(0) - \delta\dot{x}_2(0)]t_c + (a_1 - a_2)\left[1 + \beta t_c - e^{\beta t_c}\right] + (b_1 - b_2)\left[1 - \beta t_c - e^{-\beta t_c}\right] \quad (50)$$

These two equations can be solved for $(a_1 - a_2)$ and $(b_1 - b_2)$. They can then be combined with the expressions for $(a_1 + a_2)$ and $(b_1 + b_2)$ to yield the arbitrary constants a_1, b_1, a_2, and b_2. These can then be used to determine the remaining constants in the problem c_1, d_1, c_2, and d_2.

This completes the computation of the perturbations required to resolve conflicts. These perturbations can be added to the nominal paths to yield the conflict resolved trajectories. Note that the perturbation equations are solved from the current time to the final time in every guidance interval. These perturbations correct the nominal waypoints to form the new waypoint sequence. Note that a simple addition of the perturbations to the nominal may not always resolve the conflict. In highly nonlinear geometries, the conflict resolution computations will have to be repeated several times to generate conflict-free waypoint sequences. In these cases, it is beneficial to multiply the perturbations by an attenuating factor before adding to the nominal trajectories. The resulting algorithm will turn out to be the neighboring extremal technique.[18,39] The conflict resolution algorithm is next evaluated for a set of conflict scenarios involving two aircraft. These simulation results are given in the next subsection.

B. Guidance Law Simulation Results

A two-aircraft conflict scenario is chosen to demonstrate the guidance law performance. In all cases, the weighting factor β is chosen to be 10^{-3}. The conflict resolution case considered is that of two aircraft with crossing trajectories. Figure 5 shows the down-range vs cross-range trajectories, altitude profiles and interaircraft

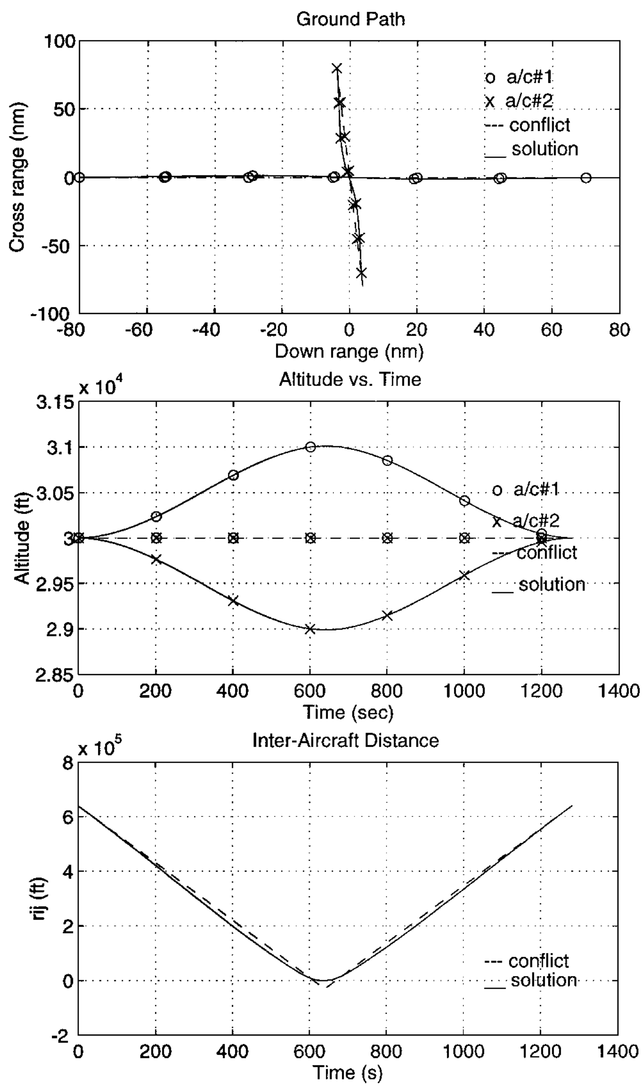

Fig. 5 Conflict resolution trajectories using the neighboring extremal guidance law: crossing trajectories.

distances before and after the conflict resolution maneuvers. The trajectories before conflict resolution are denoted by dashed lines, and the trajectories after conflict resolution are denoted by solid lines. Because cooperative conflict resolution is assumed, both aircraft perform conflict resolution maneuvers, as can be observed in the down-range vs cross-range plot. Because of the assumption that equal amounts of conflict resolution maneuvers will be made in the down-range, cross-range, and altitude directions, the aircraft also perform maneuvers in the vertical plane. It can be observed from the interaircraft distance shown in Fig. 5 that each aircraft has adjusted its trajectory to resolve the conflict.

In the present example, the guidance problem was solved as a neighboring extremal problem. In this formulation, the trajectory perturbations generated by the guidance law are attenuated before being added to the nominal trajectories. The updated nominal trajectory is then used to compute the conflict resolution trajectory perturbations. Iterative computations are continued until the conflict is resolved. In the present case, an attenuation factor of 0.2 is used, and the solution converged in three iterations.

The simulation results presented in this subsection illustrate the performance capabilities of the conflict resolution guidance law. In each case, the guidance law resolved the conflicts using modest control magnitudes. As indicated elsewhere in this section, the guidance law can be extended to include multiple aircraft executing widely different nominal trajectories. Moreover, inequality constraints can be imposed on the control magnitudes to improve the implementability of the present guidance laws. The ride quality of these conflict resolution guidance laws can be further improved by introducing control rate limits. This will produce smooth control settings, resulting in nearly jerk-free trajectories. Finally, the present formulation can be extended to the spherical Earth case. The kinematic trajectory models of the form given in Ref. 4 can be readily adapted for this purpose.

The derivations presented in this section, together with the simulation results, amply demonstrate the feasibility of developing onboard implementable guidance laws for conflict resolution in free flight.

V. Conclusions

This paper documented the results of a research study that examined development of advanced algorithms for air traffic conflict resolution. The research was motivated by the free-flight concept currently being considered for implementation as a part of the national air traffic control system. Because different aircraft in the airspace may have conflicting objectives, the desired trajectories chosen by individual aircraft can produce conflicts with other aircraft trajectories. Whereas conflict resolution strategies are obvious under low-speed, low-traffic-density conditions, the problem can become complex as the traffic density and the aircraft speeds increase.

The conflict resolution problem was formulated and solved as a multiparticipant optimal control problem. The problem was formulated under the assumption that the objective of each aircraft involved in a potential conflict is to optimize its performance while maintaining adequate separation between other aircraft in the vicinity. Point-mass aircraft models and oblate spheroidal conflict envelopes were used. The models included thrust, load factor, and bank angle as the control variables. Inequality constraints were imposed on the aircraft altitude, climb/descent rate, airspeed, and control magnitudes.

Two distinct approaches for solving the conflict resolution problem were advanced. In the first approach, the aircraft trajectories were approximated using piecewise-linear paths to convert the trajectory optimization problems into parameter optimization problems. Parameterized conflict resolution problems were solved using a single-objective SQP method, and a multiple-objective goal attainment method. Conflict resolution involving six aircraft was demonstrated. In the second approach, the aircraft equations of motion were linearized about the nominal trajectories, and a state-constrained trajectory optimization problem was formulated. This problem was then solved in closed-form to synthesize a conflict resolution guidance law. Performance of the guidance law was demonstrated for crossing aircraft.

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